CLEAR- Cahoots-Like Event at Rutgers
Math
Packet by Sam Braunfeld

If the characteristic of a field does not divide the order of a given finite group, then the group algebra is this type of structure, by Maschke's theorem. In an abelian category, the middle term of a short exact sequence is this type of structure in relation to the other nontrivial terms if and only if there exists a retract of the injection or a section of the surjection. Composition series are useful for modules not nicely expressible as this type of structure, the non-semisimple modules. A group is one of these structures if there exist two subgroups such that every element can be uniquely written as a product of two elements, one from each subgroup. For 10 points, name these structures that are frequently the Cartesian product of substructures with operations defined componentwise.
ANSWER: **direct sum** [or **direct product**]

The uniformization theorem gives the existence of one of these functions between certain objects. If the domain and image are bounded by Jordan curves, then these functions can be extended continuously to the boundary by Caratheodory's theorem. One of these functions taking the real line to the boundary of a polygon is given by the Schwarz-Christoffel integrals. A rotation times 1 minus alpha *z* over 1 minus alpha bar *z*, with modulus of alpha less than 1, characterizes this type of function from the unit disc to itself, while the fractional linear transformations give all these functions from the Riemann sphere to itself. For any simply connected, open, proper subset of C, there exists one of these functions whose image is the unit disc, by the Riemann mapping theorem. For 10 points, name these functions that preserve angles.
ANSWER: **conformal** functions [or **biholomorphic** functions; or **biholomorphism**s; accept equivalents for "functions", e.g. "maps", "mappings", "transformations"]

In the going up and going down theorems, there are two chains of prime ideals in one of these objects, one partially lying over the other, and those theorems can be used to show the Krull dimension of each part of these objects are equal. For a unique factorization domain, any of these objects generated by elements from its field of fractions is trivial. The Cayley-Hamilton theorem, or Nakayama's lemma, can be used to show that given one of these objects generated by *x* over A, there exists a faithful A adjoined *x* module that is finitely generated as an A module. That equivalence is used to prove that forming one of these objects by taking their namesake closure gives a subring. For 10 points, name these objects in which every element in one ring satisfies some monic polynomial with coefficients in the base ring.
ANSWER: **integral extension**s

For a locally compact abelian group G, this object consists of the continuous homomorphisms from G to the circle group. For an L-p space with 1 less than or equal to p less than infinity, if 1/p+1/q=1, this object is isomorphic to L-q. An isomorphism between a real Hilbert space and this space is given by the Riesz representation theorem. There is a natural embedding from any vector space into the double-type of this space, and a non-natural embedding into this space sending a basis vector *v* to the function returning 1 on *v* and 0 on any other basis vector. For the space of continuous functions on a closed interval, integration is an element of this space, since it is linear and outputs a real. For 10 points, name this space of linear forms from a vector space to its base field.
ANSWER: **dual** space [or continuous **dual** space; or **dual** group]

The product form of this technique is equivalent to performing it with each factor sequentially, which can be used to prove Easton's theorem. In this technique, the delta-systems lemma can be used to show chain conditions are satisfied, which in turn show cofinalities are preserved. This technique constructs an object consisting of the values of names relative to a generic filter of a given poset. This technique's first application used the poset of finite partial functions from omega 2 cross omega to the set 0,1, essentially adding omega 2 many new reals. For 10 points, name this technique for constructing models of set theory, such as one violating the continuum hypothesis.
ANSWER: **forcing**

In a field of characteristic p, the extension given by the splitting field of *x* to the *p* minus *x* plus *a* corresponds to one of these objects, by Artin-Schreir theory. These objects of order *n* have a representation given by a diagonal matrix with the n^{th} roots of unity on the diagonal. The automorphism group of these objects is isomorphic to their unit group. Any finitely generated abelian group can be decomposed as a direct sum of these groups. When finite, these groups have exactly one subgroup for every divisor of their order. The rotational symmetry group of a regular

polygon is this type of group. These groups are either isomorphic to Z or Z mod *n* Z for some *n*. For 10 points, name these groups generated by a single element.
ANSWER: **cyclic** groups

The zeta functions of projective varieties over these structures are the subject of the Weil conjectures. The Fano plane can be constructed as a 3-dimensional vector space over one of these structures. All automorphisms of these structures are generated by a p^{th} power mapping, the Frobenius automorphism. Every one of these structures can be generated by adjoining the roots of *x* to the *p* to the *n* minus *x*. Since 1 plus the product of *x* minus *a* over every element *a* of one of these structures has no root, they are never algebraically closed. The classification finite abelian groups can be used to prove these structures have cyclic multiplicative groups. For 10 points, name these structures, examples of which include Z mod *p* Z with addition and multiplication.
ANSWER: **finite field**s [prompt on "fields"]

Every set satisfying determinacy has the perfect set property, the Baire property, and this property. Taking the preimage of a set without this property under the function *x* plus the Cantor function of *x* yields a set with this property, but not a similar, stronger property. Sets with this property can be defined by starting with the algebra of half-open sets and the function that takes an interval a,b and returns b minus a, and then applying the Caratheodory extension theorem. Picking one representative in the interval 0,1 for each element of R mod Q gives a set without this property, the Vitali set. Sets with this property belong to the completion of the Borel sigma-algebra. For 10 points, name this property of sets of reals which have a translation-invariant, countably-additive generalization of length.
ANSWER: **Lebesgue measurability** [prompt on "measurability"]

Given a commutative ring, you can form an incidence algebra on this type of structure, one of whose elements is the structure's Mobius function. If these structures have the LYM property, then they also have the Sperner property. Dilworth's theorem gives the width of these structures as the size of the either the largest antichain or the minimal partition into chains. The morphisms between these structures are monotone functions. These structures can be pictured with Hasse diagrams. Examples of these structures include the natural numbers with divisibility, and a power set with containment. For 10 points, name these sets together with an ordering in which two elements may be incomparable.
ANSWER: **partially ordered** sets [or **poset**s; or **partial order**ings; or **lattice**s]

Every countable Borel equivalence relation can be induced by one of these functions, by the Feldman-Moore theorem. A lemma on the number of fixed points of some of these functions can be used to prove the Sylow theorems. A linear one of these functions on a vector space is a representation. Using one of these functions given by left multiplication gives a proof of Cayley's theorem. Since these functions give well-defined functions using the cosets of the stabilizer of x, there is a natural bijection between the group quotiented by the stabilizer of an element and the orbit of that element under one of these functions. For 10 points, name these functions equivalent to a homomorphism from a group to the symmetry group of a set.
ANSWER: **group action**s

Warning: Two answers required. The pronoun object1 will be used to refer to one answer, while object2 will refer to the other, but answers are acceptable in either order.
For a complete object1, the number of countable object2s cannot be 2, since if the atomic and saturated object2's are distinct then one can construct a third. If an object1 has no finite object2's and is kappa-categorical for some kappa, then it is complete, by Vaught's test. An object1 is complete, then all its object2s are elementarily equivalent. An object1 has an object2 if every finite subset of that object1 has an object2, by the compactness theorem. An object1 can be obtained from any structure by taking the sentences true in that structure, and that structure will then be an object2. A structure is an object2 if it admits an interpretation of the non-logical symbols making every sentence in an object1 true. For 10 points, name these objects, a set of sentences and a structure satisfying them.
ANSWER: first-order **theories** and **model**s

Qualitative features of these objects can be determined by checking the moduli of their Floquet multipliers. Constructing a local section through a point in one of these objects, and then mapping nearby points to their first return gives the Poincare map. One of these objects appears or disappears when a fixed point changes stability in a

Hopf bifurcation. For a planar system, a closed, bounded limit set containing no fixed points must be one of these objects, by the Poincare-Bendixson theorem. These objects cannot occur in a gradient system, since the potential must strictly decrease with time. Aside from the origin, the phase portrait for a simple harmonic oscillator consists entirely of these objects. For 10 points, name these solutions of dynamical systems that form closed curves in the phase space.
ANSWER: **periodic orbit**s [or **closed orbit**s; or **periodic solution**s; or **limit cycle**s]

Suslin's problem asks if the countable chain condition is equivalent to this property for certain dense linear orders. A Hilbert space has this property if and only if it has a countable orthonormal basis. A compact metric space can be covered by finitely many balls of radius 1 over *n*, so taking the centers of these balls for all *n* shows the space has this property. The space of continuous functions on a closed interval has this property by the Weierstrass approximation theorem, since you can approximate real polynomials by rational ones. A topological space with a countable basis has this property, since you can just pick one point in each basic open set. Since any real is the limit of a sequence of rationals, the reals have this property. For 10 points, name this property defined as having a countable, dense subset.
ANSWER: **separability**

The Calderon-Zygmund lemma states that integrable functions have this property with respect to a given constant almost everywhere outside of some open cubes. The Baire category theorem can be used to prove that for certain sets of operators on a Banach space, the pointwise form of this property implies its uniform form. If every element of a sequence of functions has this property with respect to some integrable function, then limits and integrals can be interchanged. Entire functions with this property are constant by Liouville's theorem. Uniform convergence corresponds to convergence in a metric space on functions with this property if they are given the sup norm. For 10 points, name this property possessed by continuous functions on a compact domain, by the extreme value theorem.
ANSWER: **bounded**ness

Inner models of measurability can be defined by taking this structure relative to a kappa-complete ultrafilter. As part of his fine structure theory, Jensen showed this structure satisfies the diamond property, using the fact that any transitive elementary submodel of this structure equals one of its levels. Additionally, the condensation lemma shows every bounded subset of a cardinal kappa appears by the kappath stage, so this structure satisfies GCH. Since there is a sigma 1 well-ordering of this structure, which uses Godel numbering of formulas, it satisfies the axiom of choice. For 10 points, name this minimal standard model of ZFC, defined as a hierarchy where every set in some level is a definable subset of those below it.
ANSWER: Godel's **constructible universe** [or Godel's **constructible hierarchy**; or **L**]